A New Approximate Gradient Algorithm Applied in Constrained Reservoir Production Optimization

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***Abstract*—In this paper, a new approximate gradient method is proposed for constrained reservoir production optimization. The new algorithm method is gradient-free, which is a compromised solution to finite-difference method. To get a quick evaluation of the gradient, all parameters are perturbed at one time stochastically and the calculated gradient is also stochastic. Based on the relationship between gradient and direction derivative, we construct a new search direction with the stochastically generated perturbation vector. It is proved that the stochastic gradient is always an uphill direction, ensuring that a better solution can be found along the stochastic gradient direction. Besides, projected gradient method is incorporated into the new algorithm to deal with constraints in production optimization. A comparison is made between the new algorithm and simultaneous perturbation stochastic approximation (SPSA) algorithm using a synthetic reservoir case. The results show that the new method outperforms SPSA in constrained production optimization problem. After optimizing the production strategy for a synthetic reservoir, the economic benefit improves about 20%.**

***Index Terms*—constrained reservoir production optimization, SPSA algorithm, approximate gradient, directional derivative, projected gradient method**

I. Introduction

Optimization theory [1], [2] has been introduced into reservoir decision making for tens of years. Reservoir production optimization process can be expressed as an optimization problem. By controlling production and injection operation parameters periodically, the reservoir economic benefits reach the optimal condition. Generally, both gradient-based methods and gradient-free methods have been adopted in reservoir optimization. For an optimization problem, gradient methods [3], [4] seem to be the most efficient. However, it is hard to gain and store the huge data information in calculating the gradient matrix using gradient-based methods in large-scale problems. More and more gradient-free methods take part in production optimization.

Generic algorithm (GA) [5] was an effective random search optimization method, which has been introduced into petroleum engineering field for a long time. In 2003, Yeten B., Durlofsky L.J. and Aziz K. [6] used GA in the optimization of well location, well trajectory as well as well pattern. M. tavakkolian, F. Jalali F. and M.A. Emadi [7] applied GA in optimizing the performance of a production well and concluded that GA was an influential tool for solving petroleum engineering problems. Similar to GA, particle swarm optimization (PSO) method [8] was a population based stochastic optimization technique which was proposed by Dr. Eberhart and Dr. Kennedy in 1995. Onwunalu and Durlofsky employed PSO method to the optimization of well patterns [9]. Hui Zhao, Gaoming Li and A.C.Reynolds introduced PSO into production optimization [10] and compared the effectiveness of PSO with other gradient-free methods.

Although gradient-based methods were thought to be the most effective, the huge difficulties of calculating gradient limited the application in large-scale reservoir production optimization. To overcome this default, approximate gradient methods were proposed.

With just one simultaneous stochastic perturbation, an approximate gradient was calculated. Ref. [11] proved that the SPSA gradient direction was an uphill direction and the expectation of SPSA gradient was the real gradient direction. Gao G., G. Li, and A. C. Reynolds firstly applied SPSA method in automatic history matching [12]. In 2007, Wang, C., G. Li, and A. C. Reynolds used SPSA in production optimization [13] under the context of close-loop reservoir management. Ensemble-based optimization (EnOpt) method [14] is another gradient evaluation method, developed by Chen, Y., D. Oliver, and D. Zhang. In reservoir production optimization process, the gradient was obtained by calculating the covariance between the control vector and NPV.

In this paper, a new approximate gradient method is proposed for production optimization. The new algorithm method is gradient-free. The new algorithm is a compromised solution to finite-difference method. To get a quick evaluation of the gradient, all parameters are perturbed at one time stochastically. As a result, the calculated gradient is also stochastically. Based on the relationship between gradient and direction derivative, we construct a new search direction with the stochastically generated perturbation vector. The stochastic gradient is proved to be an uphill direction, ensuring that a better

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solution can be found along the stochastic gradient direction. Besides, projected gradient method is incorporated into the new algorithm to deal with constraints in production optimization. A comparison is made between the new algorithm and simultaneous perturbation stochastic approximation (SPSA) algorithm using a synthetic reservoir case. The results show that the new method outperforms SPSA in constrained production optimization problem.

II. Mathematical Modeling

Reservoir production optimizing process can be expressed as an optimization problem with three gradients: control variables, objective functions, and constraints. By controlling wells' operation parameters periodically, certain production indexes of the reservoir reach the optimal condition. For practical oilfield production, investors focus more on economic profits, well known as net present value (NPV). The objective function of reservoir production optimization is described,

$$\max J(\mathbf{u}) = \sum_{n=1}^{T}\left[\sum_{j=1}^{N_P}(r_o q_{o,j}^n - c_{wp} q_{wp,j}^n) - \sum_{i=1}^{N_I} c_{wi} q_{wi,i}^n\right]\frac{\Delta t^n}{(1+b)^{t^n}} \quad (1)$$

where $J(\cdot)$ is the total economic profits, \$; $\mathbf{u}$ is the control variables to be optimized such as production rates, injection rates, well bottom hole pressure(BHP); T is the total number of simulation time steps; N_P and N_I are the total number of producers and water injectors respectively; r_o is the price of crude oil, \$/STB; c_w and c_{wi} are the water processing cost and injecting cost respectively, \$/STB; $q_{o,j}^n$ and $q_{wp,j}^n$ are the average oil and water production rate of the j-th producer at the n-th simulation time step, STB/day; $q_{wi,i}^n$ is the average water injection rate of the i-th injector at the n-th simulation time step, STB/day; b is the annual discount rate, %; Δt^n is the time interval of the n-th simulation time step, day; t^n is the cumulative time up to the n-th simulation time step, year.

In oil production process, operation parameters of producers and injectors are limited to the reservoir geological properties, fluid properties, as well as ground production facilities. That is to say, the maximum problem defined in (1) is subject to variable constraints. Equations (2)-(4) are corresponding to the equality constraints, non-equality constraints and boundary constraints.

$$e_i(\mathbf{x},\mathbf{u}) = 0, i = 1, 2, \cdots n_e. \quad (2)$$

$$c_j(\mathbf{x},\mathbf{u}) \le 0, j = 1, 2, \cdots n_{ne} \quad (3)$$

$$\mathbf{u}^{low} \le \mathbf{u} \le \mathbf{u}^{up} \quad (4)$$

where $\mathbf{x}$ represents the reservoir property vector (porosity, permeability, saturation and so on).

III. Optimization Method and Solution Process

A. New Approximate Gradient Estimation

In mathematics, gradient is a generalization of the usual concept of derivative to the functions of several variables. Directional gradient is the change rate of the functions along a certain directions. Therefore, directional gradient is relevant to gradient shown in (7).

$$\text{Grad J=}\begin{bmatrix}\frac{\partial J}{\partial u_1}\\ \frac{\partial J}{\partial u_2}\\ \vdots \\ \frac{\partial J}{\partial u_N}\end{bmatrix} = \begin{bmatrix}\frac{J(\boldsymbol{u}+\alpha\boldsymbol{e}_1)-J(\boldsymbol{u})}{\alpha}\\ \frac{J(\boldsymbol{u}+\alpha\boldsymbol{e}_2)-J(\boldsymbol{u})}{\alpha}\\ \vdots \\ \frac{J(\boldsymbol{u}+\alpha\boldsymbol{e}_N)-J(\boldsymbol{u})}{\alpha}\end{bmatrix}. \quad (5)$$

$$\frac{\partial J}{\partial l} = \frac{J(\boldsymbol{u}+\Delta\boldsymbol{u})-J(\boldsymbol{u})}{\|\Delta\boldsymbol{u}\|_2} \quad (6)$$

$$\frac{\partial J}{\partial l} = \frac{\partial J}{\partial u_1}\cos\theta_1 + \frac{\partial J}{\partial u_2}\cos\theta_2 + \cdots + \frac{\partial J}{\partial u_N}\cos\theta_N = (\boldsymbol{cos}\langle\boldsymbol{\theta}\rangle)^T\boldsymbol{g} \quad (7)$$

where $\boldsymbol{e}_i$ is the unit vector in i-th direction; $\Delta\boldsymbol{u}$ is a tiny disturbance at point $\boldsymbol{u}$; $\boldsymbol{l}$ represents a specific direction; θ_i is the intersection angle of $\boldsymbol{e}_i$ and $\boldsymbol{l}$.

According to simultaneous perturbation stochastic approximation (SPSA) algorithm, a vector $\boldsymbol{\Delta}$ of independently distributed random variables with an average value of zero is generated. The SPSA approximate gradient is,

$$\tilde{g} = \begin{bmatrix}\frac{J(\boldsymbol{u}+\boldsymbol{\Delta})-J(\boldsymbol{u})}{\Delta_1}\\ \frac{J(\boldsymbol{u}+\boldsymbol{\Delta})-J(\boldsymbol{u})}{\Delta_2}\\ \vdots \\ \frac{J(\boldsymbol{u}+\boldsymbol{\Delta})-J(\boldsymbol{u})}{\Delta_N}\end{bmatrix} \quad (8)$$

where Δ_i is the i-th variable value in the vector $\boldsymbol{\Delta}$.

The directional gradient at $\boldsymbol{\Delta}$ direction is,

$$\frac{\partial J}{\partial \boldsymbol{\Delta}} = \frac{J(\boldsymbol{u}+\boldsymbol{\Delta})-J(\boldsymbol{U})}{\|\boldsymbol{\Delta}\|_2} \quad (9)$$

Combining (8) and (9), the i-th component of SPSA approximate can be expressed as,

$$\tilde{g}_i = \frac{J(\boldsymbol{u}+\boldsymbol{\Delta}) - J(\boldsymbol{u})}{\Delta_i} = \frac{J(\boldsymbol{u}+\boldsymbol{\Delta}) - J(\boldsymbol{u})}{\|\boldsymbol{\Delta}\|_2} \cdot \frac{\|\boldsymbol{\Delta}\|_2}{\Delta_i} \quad (10)$$

Substituting (10) into (9), the equation becomes,

$$\frac{\partial J}{\partial l} = \tilde{g}_i \cdot \cos\theta_i \quad (11)$$

The sum of N components is,

$$n\frac{\partial J}{\partial l} = \tilde{g}_1 \cdot \cos\theta_1 + \tilde{g}_2 \cdot \cos\theta_2 + \cdots + \tilde{g}_N \cdot \cos\theta_N$$
$$= (\tilde{\boldsymbol{g}})^T \cos\langle\boldsymbol{\theta}\rangle = (\cos\langle\boldsymbol{\theta}\rangle)^T \tilde{\boldsymbol{g}} \quad (12)$$

The product of (7) and (12) is,

$$n\left(\frac{\partial J}{\partial l}\right)^2 = (\boldsymbol{g})^T \cdot \cos\langle\boldsymbol{\theta}\rangle \cdot (\cos\langle\boldsymbol{\theta}\rangle)^T \cdot \tilde{\boldsymbol{g}} \geq 0 \quad (13)$$

We define a new direction $\boldsymbol{g}$ as an approximate gradient,

$$\hat{\boldsymbol{g}} = \cos\langle\boldsymbol{\theta}\rangle \cdot (\cos\langle\boldsymbol{\theta}\rangle)^T \cdot \tilde{\boldsymbol{g}} \quad (14)$$

From (13), we know that, $\hat{\boldsymbol{g}}$ is always an uptrend.

B. Constraints Handling

1) Boundary constraints

One way to deal with the upper-lower boundary constraints is to employ log-transformation. In log space, the new derived variables are no longer subject to boundary limits. After getting the optimal value in log space, inverse log transformation is used to get the exact value in original space. The log transformation formula is,

$$s_i = \ln\left(\frac{u_i - u_i^{low}}{u_i^{up} - u_i}\right) \quad i = 1, 2, \cdots, N \quad (15)$$

where s_i is the i-th component of the variable vector in log space.

The inverse log transformation formula is,

$$u_i = \frac{\exp(s_i) \cdot u_i^{up} + u_i^{low}}{1 + \exp(s_i)} = \frac{u_i^{up} + u_i^{low} \cdot \exp(-s_i)}{1 + \exp(-s_i)} \quad (16)$$

2) Equality constraints and non-equality constraints

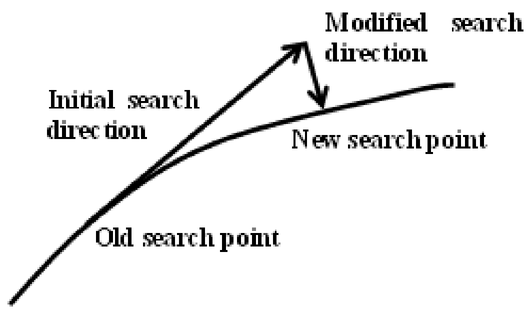


Figure 1. The illustration of modified search direction by projected gradient method.

Projected gradient method is an effective method for dealing with equality constraints and non-equality constraints, which is proposed by Rosen in 1960. The main principle of projected gradient method is illustrated in Fig. 1. If the iteration point belongs to the feasible region limited by (2)-(3), search along the gradient direction or approximate gradient direction. However, for iteration point set on the boundary line, we must make an adjustment for the search direction to pull the iteration point back to the feasible region.

The new search direction becomes,

$$\hat{\boldsymbol{g}} = \mathbf{N}\left(\mathbf{N}^T\mathbf{N}\right)^{-1}\begin{pmatrix}\Delta\mathbf{E} \\ \Delta C\end{pmatrix} + \hat{\boldsymbol{g}} \quad (17)$$

And the new iteration point can be described as,

$$\boldsymbol{u}^{\boldsymbol{k}+1} = \boldsymbol{u}^{\boldsymbol{k}} + \alpha\hat{\boldsymbol{g}} \quad (18)$$

where N is the coefficient matrix in (2)~(4); $\Delta\mathbf{E}$ is the residual vector of equality constraints; and ΔC is the residual vector of non-equality constraints; α is the search length.

C. Solution Process

The outline of the new approximate gradient method for dealing with constraint reservoir optimization is summarized as follows.

- Step 1: Give an initial value $\boldsymbol{u}_0$ for optimization.
- Step 2: Call Eclipse simulator and compute the objective function value with (1).
- Step 3: With (15), transform the variable vector in log space.
- Step 4: Calculate the approximate gradient with (14).
- Step 5: Modify the approximate gradient with (17).
- Step 6: Update the variable vector with (18).
- Step 7: Call Eclipse simulator and compute the new objective function value with (1)
- Step 8: Inverse log transform the variable vector with (16) and compare the new value with the last value. If the new function value is smaller than the last one, shorten α and repeat step 6 ~ step 7; Otherwise, update the variable vector with the new one calculated in step 6. And repeat step 2 ~ step 7 until the set search number is reached.

IV. Optimization Runs and Results

We now apply the new proposed method to a synthetic water flooding reservoir, with 25×25×1 grid blocks. The reservoir area is 106 m and thickness is 8m. Water and oil are slightly compressible fluid. The condensate water saturation is 0.3, and the residual oil saturation is 0.1. The initial pressure is 26.2MPa. From Fig. 2, we can see that the reservoir is seriously heterogeneous, with three high permeability channels.

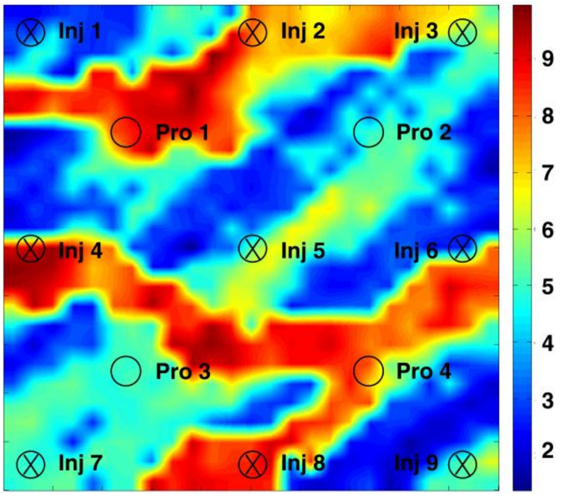


Figure 2. Log permeability distribution of the synthetic reservoir.

The reservoir contains 9 injectors and 4 producers, forming a typical five-spot well pattern. The anticipated reservoir production life is 1800 days. Suppose that the well control parameters are altered, and we have ten control steps. Therefore, the total number of control variable is (4+9)×10=130. A base production strategy is given beforehand: the oil production rate is 1400 STB/day for a single producer; the water injection rate is 345 STB/day for a corner injector, 690 for an edge well. Therefore, the total oil production rate is 5600 STB/day and the total injection rate is 5600 STB/day, which guarantees 1:1 injection-production ratio.

In the process of reservoir optimization, the total injection rate and production rate are both constant. By allotting different amount to each well, the economic benefits of reservoir production can be maximized. We set each producer rate with $0 \le q_o \le 3200$ STB/day, and each injector rate with $0 \le q_w \le 3200$ STB/day. The oil price is 104.3 $/BBL, water injection cost 13.0 $/BBL and the sewage treatment cost 13.0 $/BBL, and the annual discounting rate is 0.

In reservoir production optimization process, SPSA algorithm is an effective method. In this article, we compare our results with that of SPSA method. Main parameters used in SPSA are set as following: $\alpha = 6, A = 2, c = 0.1, a = 0.9, \gamma = 0.9$. The meaning of these constant can be found in [11]. And the initial search step used in new method in log space is 6.

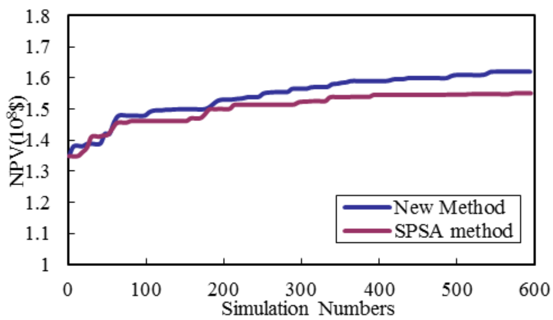


Figure 3. Optimization result of NPV with respect to simulation numbers.

According to Fig. 3, the economic profit of the initially proposed production strategy is about 1.35×10^8 \$. We optimize the production and injection parameters with our new method and SPSA method separately. After hundreds of simulation, the NPV values of the reservoir production have increased. From the final NPV's comparison, we know that the new method is more efficient than the SPSA method. The new method achieves $1.62\ \times10^8$ \$ after 600 simulations, while SPSA achieves $1.55\ \times10^8$ \$ after 600 simulations.

Fig. 4 and Fig. 5 give comparisons of the cumulative oil production and cumulative water production. Although the liquid production rate keeps constant during the process of optimization, the cumulative oil production increased, with decreasing water production. According to Fig. 4, the oil production increases from $2.5\ \times10^6$ BBL to $2.9\ \times10^6$ BBL, and the increase rate reaches 16% with new method, which is even better than SPSA method.

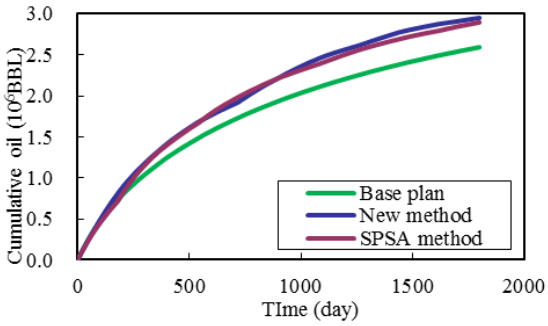


Figure 4. Relationship between cumulative oil production and time.

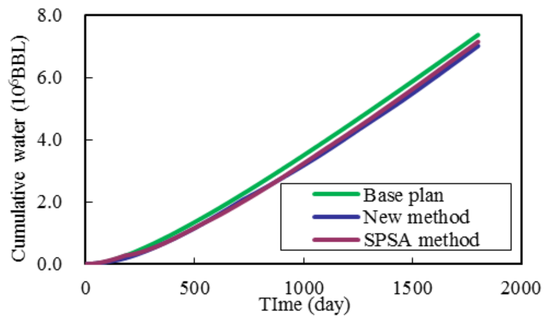
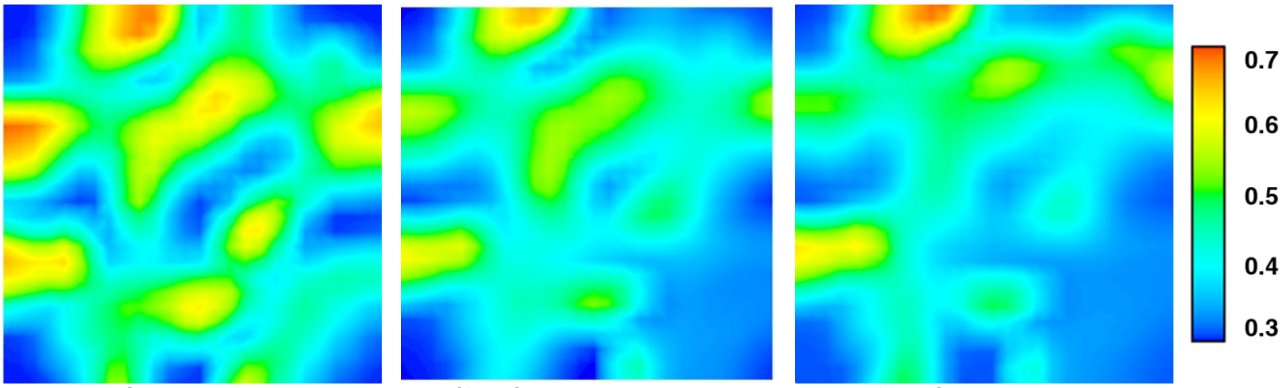


Figure 5. Relationship between cumulative water production and time.

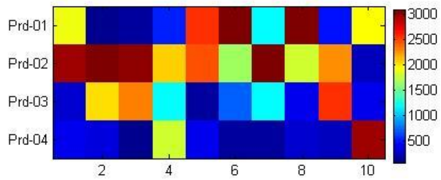
Fig. 6 show the final saturation distribution at the end of production under three strategies: base strategy A, strategy B optimized with SPSA method, and strategy C optimized with new method. Because Inj-02、Inj-05 and Inj-08 locate near the boundary of low-permeability zone and high-permeability zone, injecting water flows easily along high-permeability zone and get to Prd-01 and Prd-03 first, which forms a high-velocity water path. The viscosity of oil is bigger than that of water, therefore, injected water advances suddenly along water communication path and leaves low permeability zone undeveloped. Large part of oil remains in the reservoir as shown in Fig. 6(a) and the recovery ratio is low. However, by optimizing the control variables for each producer and each water injector, the displacement front becomes much more even, sweep efficiency becomes bigger and less oil is left in the reservoir, which can be seen in Fig. 6(b) and Fig. 6(c).



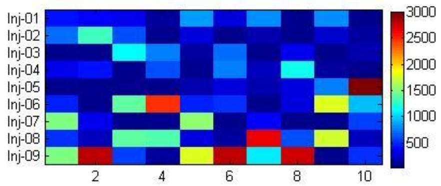
(a) production strategy A (b) production strategy B (c) production strategy C

Figure 6. Final oil saturation distribution of three production strategies.

Fig. 7 displays the production control variables for each well at each time step. We can provide some suggestions: (1) Under the capacity of facilities, increasing total water injection may make more economic profits; (2) For corner wells (Inj-1, Inj-3, Inj-7 and Inj-9), long-time water injection leads to high local pressure, which make water displacement front uneven. It is advisable to decrease the injection rate or adopt cyclic water flooding; (3) As Prd-01and Prd-03 is in the high permeability zone, high bottom hole pressure at the initial stage can improve the efficiency of water injection; (4) For Prd-02 and Prd-04, they are in the low permeability zone, improving producing pressure drop or changing producing pressure drop periodically may overcome its own defaults.



(a) Optimized production rate



(b) Optimized injection rate

Figure 7. Optimized production and injection parameters with the new proposed method

V. CONCLUSIONS

Based on the definition of gradient and directional derivative, a new method of calculating approximate gradient with a stochastic perturbation is proposed in this study. The stochastic gradient is proved to be an uphill direction, with which the maximum value can be obtained. Meanwhile, the gradient projected method is employed to deal with the effect of constraints in practical reservoir production optimization. Combined with the new proposed method and the projected gradient method, constrained production optimization is successfully solved.

A synthetic heterogeneous reservoir with high permeability streak is employed to test the efficiency of the new method. Comparing the results obtained by new method and SPSA method, the new method outperforms SPSA method. After optimization with our new method, the NPV value is improved from 1.35 $\times 10^8$ \$ to 1.62 $\times 10^8$ \$, and the cumulative oil production increases from 2.5 $\times 10^6$ BBL to 2.9 $\times 10^6$ BBL.

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