The Mathematical Modeling Research of Interest Allocation in the Third-party Automobile Logistics Partners

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***Abstract*—At present, the third party logistics in our country is developing rapidly, it also be "The third profit source" of the automobile manufacturing enterprise. Therefore, scientific, reasonable and fair profit distribution model can better encourage member enterprises to reduce logistics cost in guarantee the quality of service. Combining with the third party automobile logistics practical action, on the basis of previous studies, increasing the factor of the performance evaluation to distribution profit, we give the third party logistics enterprise and supplier performance evaluation index. The integrated use of fair theory, Shapley value method, and fuzzy comprehensive evaluation method, the dimensionless and analytic hierarchy process (AHP) to construct the profit distribution model based on four factors of "the cost, contribution, venture, performance" respectively. According to the experts estimate method giving the weight of the four factors and structuring the comprehensive profit distribution model, then list the interest distribution algorithm and the results in detail through an example.**

***Index Terms*—The third party automobile logistics, Mathematical modeling, Interest allocation**

I. Introduction

Along with the advance in science and technology, the competition of automobile industry become more and more fierce, each car manufacturers have started to save the logistics cost to reduce the manufacturing cost and seek more interests. Gradually adopting the JIT (Just In Time) and "Zero Inventory" production and management concept. The refinement of logistics operation require the distribution operation in small batch and multi-frequency. According to the advanced manufacturer's experience of the foreign automobile industry, our country automobile manufacturers and the third party logistics enterprise cooperating in the way of principal-agent contract. The third-party logistics enterprises with the automobile manufacturing enterprises as the core, in coordination with the parts suppliers, reduce the transportation costs and warehousing expenses by measures of establishing the supply-hub, optimizing the cycle pickup transport path, establishing emergency supply and distribution planning, thereby reduce the cost of the entire supply chain. For the supply chain members enterprise cooperation will produce greater income, but if the distribution of interests is not fair , reasonable and scientific , it is difficult to ensure the smooth progress of cooperation and the third party logistics enterprise's rapid reaction to the market opportunities.

Ref. [1],Shapley put forward a formula about *n* players cooperation profit distribution in 1953, this formula research the profit distribution in the aspect of quantitative. It provides a good reference value for the profit distribution, which come from the cooperation among supply chain enterprise. Ref.[2]-[4],DAI Jianhua put forward a correction algorithm on the basis of Shapley value method ,which have considered the risk factors, but have not considered the cost factors; HUA Peng and others obtained interests allocated value by using weighted alliance enterprise location model, based on the distribution of interests respectively in method of Shapley value, Nash negotiation model and the simplified MCRS; Yao Guanxin and others draw the final distribution value of interests on the basis of Shapley value method to consider three factors of "cost, contribution and risk". All of the above research did not consider the performance evaluation of the member enterprises. But every member enterprise's ability and efficiency to complete business have a significant influence on the whole supply chain. This paper create a model considering four factors of cost, contribution, risk, performance to realize the reasonable interest allocation.

II. Modeling

According to the practical cooperation of automobile manufacturing enterprises, \the third party logistics enterprise and the suppliers , profit distribution is divided into the following several parts:

$$\begin{cases} I=\alpha C+\beta\varphi_i(V)+\chi R+\delta T \\ \alpha+\beta+\chi+\delta=1 \end{cases} \tag{1}$$

Among them, αC means the part of interests distribution according to the resource costs invested by collaborators more than usual which is in order to increase the overall income of the cooperation system ; $\beta\phi_i(V)$ means the part of interests distribution according to the contribution level of collaborators which

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is in order to increase the overall income of the cooperation system; χR means the part of interests distribution according to the risk bore by collaborators which is in order to increase the overall income of the cooperation system; δT means the part of interests distribution according to the performance evaluation . Coefficient α、β、χ、δ can be determined according to the enterprise cooperation contract.

A. The Modeling of Interest Allocation Using the Fair Theory

In the supply chain system of automobile manufacturing enterprise based on the third party logistics, the manpower, material and financial resources were put into it by partners in addition for cooperation, are measured with money unified. $x_1, x_2, \cdots, x_n$ Means the cost devoted by these *n* enterprises. I Means the total revenue increased by cooperation. $v_1, v_2, \cdots, v_n$ Means the profit distributed profit for each enterprise.

$$c_i = I \cdot x_i / \sum_{i=1}^{n} x_i, i = 1,2,\cdots,c_n$$

B. The Modeling of Interest Allocation Using the Shapley Value

Shaley Value is a typical model to solve the problem of the profit distribution of multi-enterprise cooperation based on the level of corporate contribution to the Union. Suppose *n* companies participated in the cooperation, *N*={l, 2, 3,…, *n*} is a collection of all the participating companies, $S \in N$ is an alliance for *N* .*V(S)* is defined as the Union characteristic function, used to indicate the maximum benefit that *S* can achieved. The constraints of V(S) as follows ：①$V(\varphi)$=0,it means that if there is no enterprises participating in the alliance, the income of alliance is 0,this moment φ is an empty set.②$V(S)>\Sigma V(i)$,i={1,2,3… , *n*}.This is a necessary condition for the presence of coalition *S*. It means that for the enterprises in the alliance ,the gross profit is larger than the sum of the profits that every enterprise go it alone. The union revenue assigned as the following formula:

$$\phi_i(V) = \sum_{i \in S} \frac{(n-|S|)!(|S|-1)!}{n!}[V(S) - V(S-i)] \qquad (2)$$

C. The Modeling of Interest Allocation Using Fuzzy Comprehensive Evaluation Method

- Determine the evaluation factors set. There are seven risk existing in every company of the third-party logistics and supply chain system, as follows: The risk of transporting and handling goods, distribution activities, Reserves and inventory activities, account management activities, competition in the market, dissolution, information leakage. Whereby, we can determine the evaluation domain of the risk factors, $U = \{u_1, u_{2,} \cdots, u_7\}$.
- Determine the domain of evaluation grade, V={High, Higher, Medium, Low, Lower}, and every element of the evaluation is set an value respectively, V={0.1, 0.3, 0.5, 0.7, 0.9}, to indicate the corresponding relationship between evaluation domain grade and the value of the market risk.
- To evaluate single factor and set up fuzzy relation matrix R. Invite some relative experts to compose risk assessment team. It's assumed that there's five experts to evaluate the level of risk factors, count the evaluation results of all experts and converted into the range from 0 to 1, then get the fuzzy relation matrix R.
- To determine the fuzzy weight vector of the evaluation factors. Use the method of expert estimate to give each risk factor U a corresponding weight $a_i (i = 1,2,\cdots,7)$, meeting the conditions $a_i \geq 0, \sum a_i = 1$, then the weight set as follows: $A = (a_1, a_2, \cdots, a_7)$.
- Obtain the formula of benefits distribution. The result vector B of fuzzy comprehensive evaluation $B = A \circ R = (b_1, b_2, \cdots, b_5)$. Among them, " $\circ$ " is fuzzy operator, which means $b_j = \bigvee_{i=1}^{7}(a_i, r_{ij}) = \max_{1 \leq i \leq m}\{a_i r_{ij}\}, j = 1,2,\cdots,5$. Due to the sum of the components in B is not 1,so they need to be normalized to obtain the result $B' = [b'_1, b'_2, b'_3, b'_4, b'_5]$,then the risk of the enterprise is $R_i = B' \times V^T$ .The formula of profit distribution as follows: $R_M = R_i \bullet I / \sum_{i=1}^{3} R_i$

D. The Modeling of Interest Allocation Using the Dimensionless and Analytic Hierarchy Process

This paper have given the performance evaluation of suppliers in order to calculate the Numerical example, Ref. [5], as Table I. In actual operation, the performance evaluation of suppliers and the third party logistics enterprise should be based on the actual situation and discussed with other performance evaluators to make the decision.

Specific steps of interest allocation as follows:

(1) Dimensionless indicators[5].Due to some indexes are positive indicators, some are reverse and indicators are in the different unit, so they need to preprocessed in order to unify analysis.

The positive indicators preprocessed formulas are:

$$S_j(x) = \begin{cases} \frac{1}{2} + \frac{1}{2}\sin\left[\frac{\pi}{x_{j\max} - x_{j\min}}(x_j - \frac{x_{j\max} + x_{j\min}}{2})\right], x_{j\min} \leq x_j \leq x_{j\max} \\ 0(others) \end{cases} \qquad (3)$$

The reverse indicators preprocessed formulas are:

$$S_j(x) = \begin{cases} \frac{1}{2} - \frac{1}{2}\sin\left[\frac{\pi}{x_{j\max} - x_{j\min}}(x_j - \frac{x_{j\max} + x_{j\min}}{2})\right], x_{j\min} \leq x_j \leq x_{j\max} \\ 0(others) \end{cases} \qquad (4)$$

(2) Use Analytic Hierarchy Process to determine the weight of each index.

a) Build judgment matrix(P)

Suppose a layer has n indicators, X = {$x_1, x_2, \ldots\ldots x_m$},to compare their influence degree to the parent layer, then to confirm the proportion. We can ask experts to compare the relative importance between two indicators typically uses a 1-9 scale. The value and definition of P_{ij} is shown in Table II, p_{ij} represents the comparison result from x_i to x_j.

TABLE I. SUPPLIER PERFORMANCE EVALUATION INDEX

Target indicators	Level indicators	Secondary indicators
Evaluation Index System	Service levels u_1	Task completion rate (u_{11})
		Timely response rate for task (u_{12})
		Accuracy rate of task complete (u_{13})
	Information u_2	Completion rate of information changes (u_{21})
		accuracy rate of transmission (u_{22})
		Average transmission delay (u_{23})
	Manufacture u_3	The materiel average on-time delivery rate (u_{31})
		completion rate of order (u_{32})
		integrity rate of product (u_{33})

TABLE II. P_{ij} VALUE AND MEANING

The value of P_{ij}	Definition
1	Index x_i and index x_j are equal importance
3	Index x_i is moderate importance than index x_j
5	Index x_i is strong importance than index x_j
7	Index x_i is very strong importance than index x_j
9	Index x_i is extreme importance than index x_j
2, 4, 6, 8	For compromises between the above
Reciprocals of above	In comparing elements i and j ,if i is P_{ij} compared to j, then j is $1/P_{ij}$ compared to i

b) Using rad method to calculate weight

First, calculate the product of coefficients in each row from the judgment matrix: $M_i = \prod_{j=1}^{n} p_{ij} \ldots (i, j = 1, 2, \cdots, n)$. Second, obtain its Nth root: $\overline{w_i} = \sqrt[n]{M_i}$, and normalized vector $\overline{W} = \left[\overline{w_1}, \overline{w_2}, \cdots, \overline{w_n}\right]$, namely $w_i = \overline{w_i} / \sum_{i=1}^{n} \overline{w_i} \ldots (i = 1, 2, \cdots, n)$, then $W = \left[w_1, w_2, \cdots, w_n\right]$ is the eigenvectors, namely the weight.

c) Consistency test. The professor Sadie put forward the inspection formula, and he thought that if the consistency ratio is less than 0.1($CR \prec 0.1$), the weight distribution is reasonable. If not adjust the elements of judgment matrix until getting a satisfactory consistency.

In the formula, $\lambda_{\max}$ is the maximum eigenvalues of the judgment matrix P, n is order of the judgment matrix, RI is the average random consistency index of the judgment matrix,its specific values refer to Table III.

TABLE.III AVERAGE RANDOM CONSISTENCY INDEX TABLE

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

(3) Multiply the dimensionless indexes and index weight, then normalized, the interest allocation proportion is obtained.

III. NUMERICAL EXAMPLE

Suppose an automobile manufacturer signed a principal-agent contract with an third party logistics service enterprises. *A* coordinates the problems of shipments and supply batch of suppliers *B* and *C*, according to the demanded information provided by the automobile manufacturer. Now automobile manufacturer take out *1800,000 yuan*, which is additional revenue earned by supply chain based on the third party logistics, to inspire the *A*, *B* and *C*. The three companies *A,B and C* additionally input resource costs *100,000 yuan,50,000 yuan,60,000 yuan*, respectively, in order to increase the total revenue of the supply chain. It is known that if *A,B and C* input additional costs to run independently, all of them can gain *300,000 yuan*. If *A* collaborates with *B* can gain *800,000 yuan*, if *A* collaborates with *C* can gain *1000,000 yuan*, if *B* collaborates with *C* can gain *700,000 yuan*. How to distribute the incentive fee of 1800,000 *yuan*?

(1)The interest allocation based on resource costs which is more invested

$$c_A = \frac{10}{10+5+6} \times 1800{,}000 = 857143[\,yuan];$$
$$c_B = 428571[yuan];\, c_C = 514286[yuan]$$

(2)The interest allocation based on the contribution level

$$\phi_A(V) = 666{,}667[yuan];\quad \phi_B(V) = 516{,}667[yuan];$$
$$\phi_C(V) = 616{,}667[yuan]$$

(3)The interest allocation based on risk

Identify risk factors as $R_1 = 0.6,\quad R_2 = 0.45,\ R_3 = 0.55$, according to the fuzzy comprehensive evaluation method. Then

$$c_A = 675{,}000[yuan];$$
$$c_B = 506{,}250[yuan];\, c_C = 618{,}750[yuan]$$

(4) The interest allocation based on performance evaluation

Suppose that the maximum of the supplier average transmission delay is *30 minutes* according to the contract, and the minimum is *0 minutes*. The average transmission delay of supplier *B* is *10 minutes*, according to the performance evaluation. Transmission delay is reverse index and dimensionless it :

$$S_B(u_{23})=\frac{1}{2}-\frac{1}{2}sin\left[\frac{\pi}{30-0}(10-\frac{30+0}{2})\right]=0.75$$

According to the same algorithm we can get:

$$S_B(u_{21})=0.25; S_B(u_{22})=0.45.$$

Construct the matrix according to the Analytic Hierarchy Process (AHP):

$$P_{u2}=\begin{pmatrix} 1 & \frac{1}{5} & 3 \\ \frac{1}{3} & 1 & 2 \\ \frac{1}{3} & \frac{1}{2} & 1 \end{pmatrix}$$

Draw the weight set:

$$W_{u2i}=(w_{u21},w_{u22},w_{u23})=(0.2377,0.6072,0.1551).$$

In the same way, we can draw:

$$W_u=(w_{u1},w_{u2},w_{u3})=(0.5,0.2,0.3)$$

$$S_B(u_2)=S_B(u_{21})\cdot w_{u21}+S_B(u_{22})\cdot w_{u22}+S_B(u_{23})\cdot w_{u23}=0.3999$$

In the same way, we can draw:

$$S_B(u_1)=0.2786; S_B(u_3)=0.4658$$

$$S_B=\left(S_B(u_1),S_B(u_2),S_B(u_3)\right)$$

$$S'_B=W_u\cdot S_B{}^T=0.3590.$$

In the same way, we can draw:

$$S'_A=0.5967 \;\&\; S'_C=0.4638$$

Normalized:

$$S''=(0.4204,0.2529,0.3267);$$

So the interest allocation based on performance evaluation as follows:

$$T_A=756{,}720[yuan]; T_B=455220[yuan];$$

$$T_C=588{,}060[yuan];$$

(5)The final distribution of each enterprise

Suppose that experts have given the value of α、β、χ、δ were 0.1, 0.2, 0.3, 0.4, respectively. Then

$$\begin{cases} 1800{,}000=0.1C+0.2\varphi_i(V)+0.3R+0.4S \\ \alpha+\beta+\chi+\delta=1 \end{cases}$$

$$I_A=726{,}969[yuan]; I_B=503{,}633[yuan];$$

$$I_C=569{,}398[yuan]$$

IV. CONCLUSIONS

Performance evaluation factors can evaluate scientifically on the company's work efficiency and management level. It is convenient for all parties to choose partners and carry on the long-term cooperation in order to achieve the win-win effect. The article has established a scientific mathematical model about interest distribution based on the cost, contribution, risk, performance. The model consider the actual operation of the third party automobile logistics more roundly, effectively motivate member enterprises to cooperate with each other actively in order to reduce logistics cost. The mathematical model is proved that it is scientific, practical and rationality through the example calculation. Performance evaluation as a factor of interest distribution, effectively promoted the member enterprises to improve the quality of service and the efficiency of cooperation, so as to make cooperation more long and bring more revenue to the enterprises.

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