A Framework for Developing Boolean Set Implicit Blends from an Existing Union or Intersection Blend in Soft Object Modeling

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***Abstract*—In implicit surface, soft object modeling is getting popular because a complex soft object is able to be constructed easily by performing a soft (union) blend on primitive soft objects. Especially soft blend has lower computing complexity than other existing implicit blends because it dose addition operation only. However, in soft object modeling existing implicit blends still do not provide intersection blend containing blending range parameters for generating sequential blends, and precisely only intersection and union blends are developed. In order to solve this problem, this paper proposes two frameworks that can transform an existing union or intersection blend into a new family of Boolean set blending operations, including union, intersection, and difference blends. Based on the proposed frameworks, this paper has transformed scale function, developed from the scale method for an intersection blend, and soft blend, respectively, into two new groups of Boolean set blending operations with blending range parameters. The newly proposed Boolean set operations not only offer blending range parameters for blending range control but also have lower computing complexity and they especially can do bulge elimination in a union blend.**

***Index Terms*—soft object, implicit surface, blending operations, boolean set operations**

I. INTRODUCTION

An implicit surface is defined as a level surface of a defining function of 3D position. Furthermore, a complex implicit surface is constructed like block-building game from some primitive surfaces, such as planes, sphere [1], super-ellipsoids [2]-[3], star-solids [4]-[5], and sweep objects [6]-[7] through sequential blending operations. This is why implicit surface modeling is attracting much attention. In fact, blending operations play an important role in creating a complex implicit surface because they are able to connect primitive implicit surfaces smoothly with automatically generated transitional surface. The literature of blending operations is reviewed as follows:

(a). Pure Boolean set operations [8], *Max*/*Min*, have C^0 continuity, so they always generate non-smooth blending surfaces. Super-ellipsoidal blends [8] offer Boolean set operations with high-order continuity, C^n, $n \geq 1$. But, they deform blended primitives entirely.

(b). To make blended primitives deform locally after blending, blending operations with blending range control were proposed in [9]-[19]. They provide blending range parameters to adjust the size of the transitional surface of the resulting blending surface, so blended primitives are able to deform locally after blending.

(c). To reduce the computing complexity, soft object modeling was also proposed. Because soft objects are defined as a level surface of a non-negative field function, they can be blended easily by performing addition only, called soft blend, for a union. Existing field functions in the literature can be found in [1]-[2], [20]-[22]. In addition, set operations were proposed [23] for blending soft objects, but they do not offer blending range parameters. Therefore, union and intersection blends with blending range parameters were proposed [9], [13], [22] for local blending on soft objects.

Soft object modeling has lower computing complexity in blending than other implicit modeling techniques, however it still faces a problem that so far no difference blend with blending range parameters has been developed. Although Perlin's set operations [23] offer a difference blend, they do not provide blending range parameters. To solve the problem, this paper:

(a). Proposes a framework that can transform an existing intersection blend into a new set of Boolean set operators for blending soft objects.

(b). Proposes a framework that can transform an existing union blend into a new set of Boolean set operators for blending soft objects.

(c). Applies scale function as an intersection blend [13] and soft blend as a union blend, respectively, into the proposed frameworks. As a result, two new sets of Boolean set operators are created. Especially, the set created from the former blend offers blending range parameters and the set from the latter one has lower computing complexity.

The rest of the paper is organized as follows. Soft object modeling is reviewed in Section II. The proposed frameworks are presented in Section III. Based on the frameworks in Section III, Boolean set operators with blending range parameters are derived in Section IV. Conclusion is given in Section V.

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II. Soft Object Modeling

This section reviews soft object modeling.

A. Definition of a Soft Object

A primitive soft object is defined using a primitive defining function $f_i(v)$ by the point set

$$\{v\in R^3 \mid f_i(v)\geq 0.5\},\ i=1,2,\ldots,$$

which is also represented as $f_i(v)\geq 0.5$ and whose boundary surface (shape) is represented as a level surface by $f_i(v)=0.5$ or $f_i=0.5$ for short in this paper. Besides, symbol v stands for a point in R^3 in this paper.

For soft objects, defining function $f_i(v)$ above is required to be a non-negative field function. A field function is required to map R^3 to [0, 1] and is usually written as composition of $P(d)$ and $d_i(v)$ by

$$f_i(v)=(P\circ d_i)(v)=P(d_i(v)),$$
$$d_i(v)=L/I_r,$$

where

(a). $d_i(v)$: $R^3\to R_+$ is called distance function, where R_+ stands for $\{x\in R\,|x\geq 0\}$ in the paper. As shown in Fig. 1(a), L is the shortest distance from point v to the center c of a soft object $f_i(v)\geq 0.5$. I_r is the influential radius of $d_i(v)=1$ with respect to v, and it is the distance from center c to point I, the intersecting point of vector $v=[x,y,z]$ with the boundary of the influential region $d_i(v)\leq 1$.

(b). $P(d)$: $R_+\to[0,1]$ is called potential function which decreases to zero as the value of d increases from 0 to 1, as shown in Fig. 1(b). As a result, the value of $f_i(v)=P(d_i(v))$ decreases from 1 to 0 as the value of $d_i(v)$ increases from 0 to 1, i.e. as the value of L increases from 0 to the influential radius I_r. Some special requirements for a field function are presented in [1], [20]-[22] for blending range control.

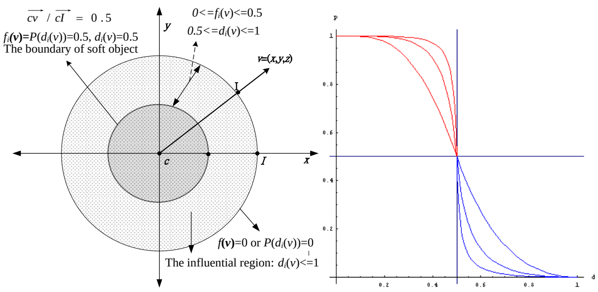


Figure 1. (a). Definition of a distance function $d_i(v)$. (b).Shapes of a potential function $p=P(d)$ as the value d increases from 0 to 1.

B. Blending Operations on Soft Objects

In order to create a more complex soft object, blending operations, i.e. implicit blends, were also proposed. An implicit blend can smoothly connect k primitive soft objects $f_1(v)\geq 0.5$,..., and $f_k(v)\geq 0.5$ via a blending operator $B_k(x_1,...,x_k)$ and it is written by

$$\{v\in R^3\,|B_k(f_1(v),...,f_k(v))\geq 0.5\},$$

which is denoted as $B_k(f_1,...,f_k)\geq 0.5$ in the following. Surface $B_k(f_1(v),...,f_k(v))=0.5$ is called blending surface.

Some existing blending operators are reviewed as follows:

(a). Soft blend (union blend) B_{Sk}:$[0,1]^k\to[0, k]$ [1]:

$$B_{Sk}(x_1,\ldots,x_k)=x_1+x_2+\ldots+x_k; \quad (1)$$

Because gradiets $\nabla f_i(v)$, i=1 to k, are zeros where $f_i(v)$ =0, $B_{Sk}(f_1,\ldots,f_k)$=0.5 have a smooth nuion blend surface.

(b). Super-ellipsoidal union and intersection blends B_{Uk}: $R_+^k\to R_+$ and B_{Ik}:$R_+^k\to R_+$ [8]:

$$B_{Uk}(x_1,\ldots,x_k)=(x_1^p+\ldots+x_k^p)^{1/p},\ \text{and}$$
$$B_{Ik}(x_1,\ldots,x_k)=(x_1^{-p}+\ldots+x_k^{-p})^{-1/p}$$

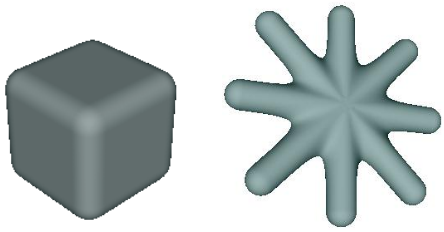
where p is a curvature parameter to adjust the shape of the transition of the resulting blending surface.

(c). Scale function $B_{A2}(x_1,x_2)$:$R_+^2\to R_+$ with blending range parameters r_1 and r_2 and a curvature parameter p for an intersection blend, from the scale method [13], is written by

$$B_{A2}=\begin{cases}(-b\pm(b^2-4ac)^{0.5})/(2a) & \text{if } a\neq 0 \text{ and } (x_1,x_2)\in\text{region III}\\ -c/b & \text{if } a=0 \text{ and } (x_1,x_2)\in\text{region III}\\ Min\ (x_1,x_2) & \text{otherwise}\end{cases} \quad (2)$$

where $a=(1+2r_2)r_1^2+(1+r_1)^2r_2^2+2p$, $b=-2(x_2(1+r_2)r_1^2+x_1(1+r_1)r_2^2+(x_1+x_2)p)$, and $c=r_1^2x_2^2+r_2^2x_1^2+2px_1x_2$. In addition, the value of B_{A2} must lie in $[h_1, h_2]$, h_1= $Min(x_1,x_2)$ and $h_2=(r_1 x_2+r_2x_1)/(r_1+r_2+r_1r_2)$, and region **III** is defined by $\{(x_1,x_2)|x_1/(1+r_1)<x_2<(1+r_2)\ x_1\}$. Moreover, scale function for a union blend can also be found in [13].

In fact, a blend $B_k(f_1,...,f_k)\geq 0.5$ is allowed to further be reused as a new primitive in other blends to generate sequential blends. For example, Fig. 2(a) shows a cube which is created using sequential intersection blends of three pairs of parallel planes by $B_2(B_2(f_1(v),f_2(v)),f_3(v))\geq 0.5$; Fig. 2(b) displays sequential union blends of four cylinders $f_i(v)\geq 0.5$, i=1,2,3 and 4, by $B_2(B_2(B_2(f_1(v),f_2(v)),f_3(v)),f_4(v))\geq 0.5$ with bulge elimination [13].



(a) (b)

Figure 2. . (a). Sequential intersection blends of three pairs of parallel planes. (b). Sequential union blends of four cylinders with bulge elimination.

III. The Framework for Developing Boolean Set Operations

In addition to those implicit blends stated in Section II, Perlin also proposed Boolean set operations for soft object modeling [20], which are written by:

(a). Intersection: $B_{I2}(x_1, x_2)=x_1x_2$.

(b). Union: $B_{U2}(x_1, x_2)=x_1+x_2-x_1x_2$.

(c). Difference of x_1 from x_2: $B_{D2}(x_1, x_2)=x_1-x_1x_2$.

A. Requirement for a Blending Operator to Develop Boolean Set Operators from Its Complement

From Perlin's Boolean set operators above, it is obtained that:

If the range of a blending operator $B_k(x_1,...,x_k)$ lies completely in the interval [0, 1], then:

(a).The complement of a soft object $f_i(v)\geq 0.5$ is given by $f_i(v)\leq 0.5$, equivalent to $1-f_i(v)\geq 0.5$ because the range of a field function $f_i(v)$ always lies completely in the interval [0, 1]. This implies that $1-f_i(v)$ can be used as a defining function of the complement of a soft object $f_i(v)\geq 0.5$.

(b).The defining function of the complement of a blending operation $B_k(f_1,...,f_k)\geq 0.5$ can be given by $1-B_k(f_1,...,f_k)$ and $B_k(f_1,...,f_k)\geq 0.5$ can be reused as a new blended primitive in sequential blends because the range of $B_k(f_1,...,f_k)$ lies in the interval [0, 1].

(c). Suppose $B_k(f_1,...,f_k)$ is an intersection, then it can derive a new family of Boolean set operations from its complement $1-B_k(f_1,...,f_k)$ and dual form by

(1). Intersection blend: $B_k(f_1,...,f_k)$.
(2). Union blend: $1-B_k(1-f_1,...,1-f_k)$.
(3). Difference blend : $B_k(f_1,1-f_2,...,1-f_k)$.

These above also prove how Perlin's set operators are derived from the complement of $B_{I2}(f_1, f_2)$ as follows:
(1). Intersection: $B_{I2}(f_1, f_2)=f_1f_2$.
(2). Union: $B_{U2}(f_1, f_2)=1-B_{I2}(1-f_1, 1-f_2)=f_1+f_2-f_1f_2$.
(3). Difference: $B_{D2}(f_1, f_2)=B_{I2}(f_1, 1-f_2)=f_1-f_1f_2$.

However, some bending operator's value might be over 1, such as soft blend $B_{Sk}(x_1,\dots,x_k)$ in (1). In addition, their domains might be defined on non-negative space, so the value of the complement $1-B_{Sk}(x_1,\dots,x_k)$ of a soft blend may be less than 0 and outside of their domain, which might cause a computational problem. Thus, a framework is proposed in the following two subsections to avoid the problem, and in particulr it can be used to develop a new family of Boolean set operators from the complement of an existing blending operator.

B. Framework for Developing Boolean Set Operators from an Intersection Operator

Step (1):

Obtain an intersection operator $B_{Ik}(x_1,\dots,x_k):R_+^{\,k}\rightarrow R_+$.

Step (2):

If the value of $B_{Ik}(1,\dots,1)$ is less than 1, then develop an increasing normalization function $N_S(u)$ which satisfies the following conditions: $N_S(0)=0$, $N_S(0.5)= 0.5$ and $N_S(B_{Ik}(1,\dots,1))=1$.

Step (3):

Since operator $N_S(B_{Ik}(x_1,\dots,x_k))$ maps $[0,1]^k\equiv[0,1]\times\dots\times[0,1]$ into interval [0, 1], according to Subsection *A*, a new family of Boolean set operations is developed from the complement and dual form of $N_S(B_{Ik}(f_1,\dots,f_k))$ as follows:

(1). Intersection blend: $N_S(B_{Ik}(f_1,\dots,f_k))$.
(2). Union blend: $1-N_S(B_{Ik}(1-f_1,\dots,1-f_k))$.
(3). Difference blend of soft object $f_1\geq 0.5$ from soft objects $f_i\geq 0.5$, i=2,...,k: $N_S(B_{Ik}(f_1, 1-f_2, \dots, 1-f_k))$.

The reasons why conditions of **Step (2)** need to be satisfied are explained below:

(a).Condition $N_S(0.5)=0.5$ ensures that blending surface $N_S(B_{Ik}(f_1,\dots,f_k))=0.5$ is always the same as the original blending surface $B_{Ik}(f_1,\dots,f_k)=0.5$ after normalization.

(b).If $B_{Ik}(1,\dots,1)$ is less than 1, then $1-B_{Ik}(1,\dots,1)$ is larger than 0, which implies that the value of newly derived union operation $1-N_S(B_{Ik}(1-f_1,\dots,1-f_k))$ may be larger than 0 at $(f_1,\dots,f_k)=(0,...,0)$. It causes that union blend $1-N_S(B_{Ik}(1-f_1,\dots,1-f_k))$ might enlarge other blended primitives in non-blending region when performing a soft blend with other soft objects. This explains why condition $N_S(B_{Ik}(1,\dots,1))=1$ for $N_S(u)$ must be satisfied if $B_{Ik}(1,\dots,1)$ is less than 1.

C. Framework for Developing Boolean Set Operators from a Union Operator

It includes three steps as follows:

Step (1): Obtain

A union operator $B_{Uk}(x_1,\dots,x_k):R_+^{\,k}\rightarrow R_+$.

Step (2):

If the value of $B_{Uk}(1,\dots,1)$ is larger than 1, then develop an increasing normalization function $N_A(u)$ which satisfies the following conditions: $N_A(0)=0$, $N_A(0.5)=0.5$, and $N_A(u)=1$ and $N_A^{(u)}(u)=0$ for all $u\geq 1$.

Step (3):

Since $N_A(B_{Uk}(x_1,\dots,x_k))$ maps $[0,1]^k$ into interval [0, 1], then according to Subsection *A*, a new set of Boolean set operations is obtained from the complement and dual form of $N_A(B_{Uk}(x_1,\dots,x_k))$ as follows:

(1). Intersection blend: $1-N_A(B_{Uk}(1-f_1,\dots,1-f_k))$.
(2). Union blend: $N_A(B_{Uk}(f_1,\dots,f_k))$.
(3). Difference blend of object $f_1\geq 0.5$ from objects $f_i\geq 0.5$, i=2,...k: $1-N_A(B_{Uk}(1-f_1, f_2,\dots,f_k))$.

The reasons why conditions of *Step (2)* need to be satisfied are explained below:

(a).Condition $N_A(0.5)=0.5$ ensures that union blending surface $N_A(B_{Uk}(f_1,\dots, f_k))=0.5$ is still the same as the original blending surface $B_{Uk}(f_1,\dots,f_k)=0.5$ after normalization,

(b).If the value of $B_{Uk}(1,\dots,1)$ is larger than 1, then $1-B_{Uk}(1,\dots,1)$ is less than 0. As a result, the value of newly derived intersection $1-N_A(B_{Uk}(1-f_1,\dots,1-f_k))$ may be less than 0 at (0,...,0), and hence intersection blend $1-N_A(B_{Uk}(1-f_1,\dots,1-f_k))$ might shrink other primitives in non-blending regions when performing a soft (union) blend with other soft objects. Regarding this, condition: $N_A(u)=1$ holds for all $u\geq 1$ can prevent intersection blend $1-N_A(B_{Uk}(1-f_1,\dots,1-f_k))$ from shrinking other blended primitives when $1-N_A(B_{Uk}(1-f_1,\dots,1-f_k))$ is reused as a new primitive in a soft blend. Besides, condition: $N_A^{(u)}(u)=0$ hold for $u\geq 1$ ensures that gradient $\nabla(N_A(B_{Uk}(f_1,\dots,f_k))$ are zeros where points are on surface $N_A(B_{Uk}(f_1,\dots,f_k))=1$ and hence gradient $\nabla(1-N_A(B_{Uk}(1-f_1,\dots,1-f_k)))$ are zeros where points are on surface $1-N_A(B_{Uk}(1-f_1,\dots, 1-f_k))=0$. As a result, when intersection blend $1-N_A(B_{Uk}(1-f_1,\dots,1-f_k)))$

or difference blend $1\text{-}N_A(B_{Uk}(1\text{-}f_1, f_2,\ldots,f_k))$ is reused as a new primitive in a soft blend, they have a smooth union blending surface with other blended primitive.

IV. Normalization Functions

Based on the conditions of the framework in Section III, this section proposes two normalization functions, respectively, for an intersection blend and a union blend.

A. Normalization Function for an Intersection Operator

Following the conditions stated in **Step (2)** in Subsection *B* of Section III, a normalization function $N_S(u)$ that maps [0, *m*], where $m=B_{Ik}(1,\ldots,1)<1$, to [0, 1] is proposed as follows:

$$N_S(u)=\begin{cases} A(u-0.5)^2+u & 0.5<u\le m \\ u & u\le 0.5 \end{cases}, \quad (3)$$

where *A* is $(1\text{-}m)/(m\text{-}0.5)^2$ and $N_S^{(u)}(u)$ is continuous at 0.5. Consequently, $N_s(u)$ is differentiable in R_+.

Because scale function $B_{A2}(x_1,x_2)$ in (2) is an intersection blend and the value of $B_{A2}(1, 1)$ is less than 0, according to the framework applying $N_S(u)$ in (3) on $B_{A2}(x_1,x_2)$ by $N_S(B_{A2}(x_1,x_2))$ gives a new set of Boolean set operations with blending range parameters r_1 and r_2 and a curvature parameter *p*, called scale set operations, as follows:

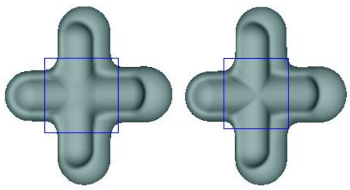
(a). Scale union blend: $1\text{-}N_S(B_{A2}(1\text{-}f_1, 1\text{-}f_2))$.
(b). Scale intersection blend: $N_S(B_{A2}(f_1, f_2))$.
(c). Scale difference blend: $N_S(B_{A2}(f_1, 1\text{-}f_2))$.

where $B_{A2}(x_1, x_2)$ and $N_S(u)$ are defined in (2)-(3).

Scale set operations provide blending range parameters, and they not only provide a difference blend but also can perform bulge elimination in a scale union blend by replacing blending range parameters r_i in $B_{A2}(x_1, x_2)$ with

$$r_i= r_i(1\text{-}\cos\theta+\omega),\ \ i=1 \text{ and } 2,$$

where r_i <0.5 and $\omega\approx 0$ and θ is the angle between the gradients of $f_1(v)$ and $f_2(v)$.



(a) (b)

Figure 3. Objects created by a scale difference of large crossing cylinders from small one by $1\text{-}N_S(B_{A2}(1\text{-}f_1, 1\text{-}f_2))$: (a). Without performing bulge elimination. (b). with bulge elimination on the marked region.

For example, Fig. 3(a) shows that a scale union of a small pair of crossing cylinders, $S_1=1\text{-}N_S(B_{A2}(1\text{-}f_1,1\text{-}f_2)=0.5$, is subtracted from a scale union of a large pair of crossing cylinders, $S_2=1\text{-}N_S(B_{A2}(1\text{-}f_3, 1\text{-}f_4))=0.5$, via a scale difference $N_S(B_{A2}(S_2, 1\text{-}S_1))$ without performing bulge elimination. Fig. 3(b) shows an object like that in Fig. 3(a), but it performs bulge elimination on the marked square region.

B. Normalization Function for a Union Operator

Following the conditions stated in **Step (2)** in Subsection *C* of Section III, a normalization function $N_A(u)$ that satisfies the conditions: $N_A^{(u)}(1)=0$, $N_A^{(u)}(0.5)=1$, and $N_A^{(u)}(u)=0$ for all $u\ge 1$, is proposed as follows:

$$N_A(u)=\begin{cases} 1 & u>1 \\ 1-4u(u-1)^2 & 0.5\le u\le 1 \\ u & u<0.5 \end{cases}, \quad (4)$$

It is easy to prove that $N_A(u)$ is continuous and differentiable at 1 and 0.5. Consequently, $N_A(u)$ is differentiable in R_+.

Because soft blend $B_{Sk}(x_1,\ldots,x_k)=x_1+x_2+\ldots+x_k$ in (1) is a union operator and the value of $B_{Sk}(1,\ldots,1)=k$ is greater than 1, according to the framework applying $N_A(u)$ in **(4)** on $B_{Sk}(x_1,\ldots,x_k)$ by $N_A(B_{Sk}(x_1,\ldots,x_k))$ gives a new set of Boolean set operations, called soft set operations, as follows:

(a). Soft intersection blend: $1\text{-}N_A(B_{Sk}(1\text{-}f_1,\ldots,1\text{-}f_k))=1\text{-}N_A(k\text{-}f_1\text{-}\ldots\text{-}f_k))$.
(b). Soft union blend: $N_A(B_{Sk}(f_1,\ldots,f_k)=N_A(f_1+\ldots+f_k)$.
(c). Soft difference blend of soft objects $f_1\ge 0.5$ from $f_i\ge 0.5$, i=2,...k: $1\text{-}N_A(B_{Sk}(1\text{-}f_1, f_2,\ldots,f_k)) = 1\text{-}N_A(k\text{-}f_1+f_2+\ldots+f_k))$.

Fig. 4 sketches the shapes of binary soft set operators: $1\text{-}N_A(B_{S2}(1\text{-}x_1,1\text{-}x_2))=0.5$, $N_A(B_{S2}(x_1,x_2))=0.5$ and $1\text{-}N_A(B_{S2}(1\text{-}x_1,x_2))=0.5$. Their shapes explain why soft set operators can be used as Boolean set operators and blend soft objects smoothly.

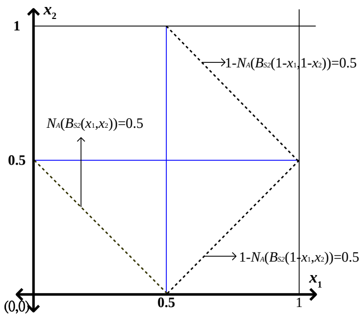


Figure 4. The shapes of the blending curves of binary soft set operators: $1\text{-}N_A(B_{S2}(1\text{-}x_1,1\text{-}x_2))=0.5$, $N_A(B_{S2}(x_1,x_2))=0.5$ and $1\text{-}N_A(B_{S2}(1\text{-}x_1,x_2))= 0.5$.

Fig. 5 demonstrates a die created via a soft difference $1\text{-}N_A(B_{Sk}(1\text{-}f_4, f_5,\ldots,f_{25}))\ge 0.5$, where $f_4=1\text{-}N_A(B_{Sk}(1\text{-}f_1, 1\text{-}f_2, 1\text{-}f_3))\ge 0.5$ is a cube like that in Fig. 2(a) and f_4 is defined by a soft intersection on 3 pairs of parallel planes, and $f_5\ge 0.5$,…, and $f_{25}\ge 0.5$ are 21 balls.

Figure 5. A die defined by a soft difference $1\text{-}N_A(B_{Sk}(1\text{-}f_4, f_5,\ldots,f_{25}))$, where $f_4=1\text{-}N_A(B_{Sk}(1\text{-}f_1,\ldots, 1\text{-}f_3))$ is a cube defined by soft intersection and $f_5,\ldots,$ and f_{25} are balls.

Here is an example demonstrating combination of scale and soft set operations. Fig. 6 displays a complex soft object which is a soft union $N_A(B_{S3}(f_1,f_2,f_3))$ of sequential scale difference operations f_1 and 2 cylinders f_2 and f_3. f_1 is given by $N_S(B_{A2}(N_S(B_{A2}(f_4, 1\text{-}f_5)), 1\text{-}f_6))$, which represents a large ball f_4 subtracted by a small ball f_5 and a cylinder f_6 sequentially.

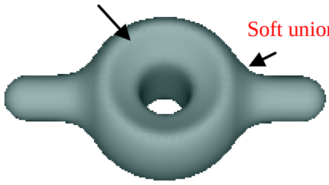


Figure 6. Combination of scale and soft set operations, performing a soft union of sequential scale difference operations with two cylinders

V. CONCLUSION

In this paper, two frameworks have been proposed to transform an existing union or intersection blend into a new set of Boolean set blending operations, including union, intersection, and difference, for soft object modeling. Based on the proposed frameworks, two new sets of Boolean set operations for soft object modeling, called scale set operations and soft set operations, have been developed by using scale function and soft blend. Especially, scale set operations offer blending range parameters and allow bulge elimination in a difference or a union blend, and soft set operations have lower computing complexity. More importantly, a new difference operation with blending range parameters has been created, and hence soft object modeling has a complete set of Boolean set operations for generating sequential blends.

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